Transient analysis of the behaviour of grounding systems consisted by driven rods

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Abstract: - Grounding systems in general are of major importance. This is not only because they provide a very low resistance path for the transient currents toward the earth but also due to the fact that they insure a nearly constant potential level for all electric and electronic appliances. The usual grounding systems consist of one or more driven rods connected with one another, horizontal rods, grounding mats, two or three dimensional grids, foundation grounding systems or even a combination of the mentioned systems when extremely low resistance paths are required.

This behaviour is evaluated by the impulse impedance of the system varying across time. That way the efficiency of the system is evaluated across the whole time that the phenomenon takes place resulting to a more accurate prediction of its behaviour. The prediction of the behaviour of such systems is thus necessary when those are imposed to transient currents such as lightning or internal faults. Transient currents are usually mathematically modelled by a double exponential function or a triple exponential if more accuracy is demanded. The grounding systems been studied are one driven rod and three driven rods connected in parallel and forming an equilateral triangle.

The systems are dealt as a combination of R, L, C elements. A mathematical analysis follows regarding the elements as compound and assuming that, in the case of more than one electrodes, the current is equally divided amongst them. The values of the R, L, C elements are computed by formulas derived from field analysis for each system.

Given the mathematical equation of the current, using network analysis, the potential difference between the striking point and remote earth is computed in the frequency domain using Laplace transformation. Inverse transform is then applied giving the equation of the potential difference in the time domain. From that point on, the calculation of the transient resistance versus time is merely a case of simple division. An alternative way of solving the problem suggests that we work in the frequency domain only using Laplace transform for the equation which describes the current. The transient resistance of the system is calculated as a function of frequency and then a inverse transform is applied to give the equation as a function of time. The transient resistance is obtained by the above mentioned analysis for the single driven rod and the three connected rods versus time. The impulse impedance is then depicted on a diagram versus time on which the steady state resistance is shown as well. The diagrams show clearly the high resistivity values for the first μsec decreasing by the time to the steady state levels for infinite time. The outcome clearly supports the opinions of the authors who state that the behaviour of grounding systems can be evaluated using mathematical models instead of making experiments which are time and money consuming.

Key-Words: - Grounding system, transient impedance, impulse current, impulse impedance, driven rod(s)

1 Introduction

The grounding systems serve multiple purposes. Not only they do insure a reference potential point for the electric and electronic devices but also provide a low resistance path for fault currents into the earth. Such fault currents can arise either from internal sources or from external ones e.g. by lightning strokes and industrially-generated static electricity. The resistance of grounding systems has an essential influence on the protection of the grounded system. Grounding systems can consist of one or more vertical or horizontal driven rods, three or more vertical driven rods connected to each other, two or three-dimensional grids from metal rods and foundation grounding systems.

The behaviour of the grounding system under lightning determines the degree of protection provided. This makes obvious the purpose of analysis procedures predicting the transient response of grounding systems. If an equivalent circuit approach is adopted these procedures can be implemented in a simulation model [1-7].

2 Fundamentals

2.1 Single vertical rod

Impulse currents (Fig. 1) are usually mathematically modelled by a double exponential function:

$$i(t) = I_o \cdot \left(e^{-a \cdot t} - e^{-b \cdot t}\right) \quad (1)$$

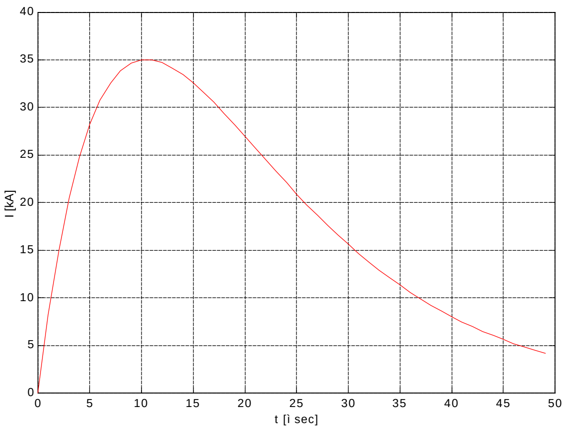

Fig. 1: Typical current waveform mathematically simulated by a double exponential equation

A simple grounding system consisting of a single vertical rod is shown in Fig.2.

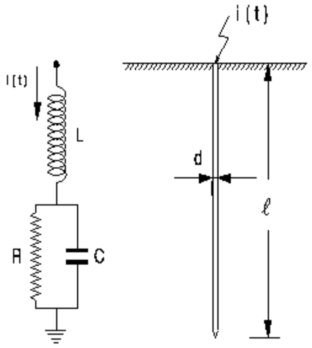

Fig. 2: Equivalent circuit for a driven rod consisting of compound elements

When this system is struck by an impulse current it appears to have a low resistance way to earth with its value decreasing exponentially to that of steady state. The differential equations which describes the potential difference between the striking point and reference earth is:

$$u(t) = R \cdot i_R + L \cdot \frac{di}{dt} \quad (2)$$

Therefore the potential difference expressed in the frequency domain is:

$$u(s) = \left[\frac{1}{C \cdot (s+a) \cdot (s+t)}\right] - \\ - \left[\frac{1}{C \cdot (s+b) \cdot (s+t)} + \left(\frac{L \cdot s}{s+a} - \frac{L \cdot s}{s+b}\right)\right] \quad (3)$$

where

$$t = \frac{1}{R \cdot C} \quad (4)$$

By applying inverse Laplace transform we obtain the expression in the time domain:

$$u(t) = I_o \cdot R \cdot \left[t \cdot \frac{e^{-t \cdot t} \cdot (b-a)}{(t-a) \cdot (t-b)}\right] + \\ + I_o \cdot R \cdot \left[t \cdot \frac{e^{-a \cdot t} \cdot (t-b) - e^{-b \cdot t} \cdot (t-a)}{(t-a) \cdot (t-b)}\right] + \\ + I_o \cdot L \cdot \left(b \cdot e^{-b \cdot t} - a \cdot e^{-a \cdot t}\right) \quad (5)$$

The impulse impedance is defined as the ratio of the impulse voltage (Eq.5) to the impulse current (Eq.1):

$$z(t) = \frac{u(t)}{i(t)} \quad (6)$$

The mathematical equation which describes the behaviour of the specific system derives from the circuit analysis regarding the elements as compound and is the following one:

$$z(t) = R \cdot \left[t \cdot \frac{(t-b) \cdot e^{-a \cdot t} - (t-a) \cdot e^{-b \cdot t}}{(t-a) \cdot (t-b) \cdot \left(e^{-a \cdot t} - e^{-b \cdot t}\right)}\right] + \\ + R \cdot \left[\frac{(b-a)}{(t - a) \cdot (t - b)} \cdot \frac{e^{-t \cdot t}}{\left(e^{-a \cdot t} - e^{-b \cdot t}\right)}\right] + \\ + L \cdot \left[\frac{b \cdot e^{-b \cdot t} - a \cdot e^{-a \cdot t}}{e^{-a \cdot t} - e^{-b \cdot t}}\right] \quad (7)$$

The stationary resistance of a driven rod is given by the following formula, assuming uniformity of current along its length [4]:

$$R = \frac{r}{2 \cdot p \cdot \ell}\left[ln\left(\frac{8 \cdot \ell}{d}\right) - 1\right] \qquad (8)$$

The capacitance of a driven rod is given by the formula [4]:

$$C = \frac{2 \cdot p \cdot e_r \cdot e_o \cdot \ell}{ln\left(\frac{4 \cdot \ell}{d}\right)} \qquad (9)$$

The inductance of a driven rod is given by [4]:

$$L = 2 \cdot 10^{-7} \cdot \ell \cdot ln\left(\frac{4 \cdot \ell}{d}\right) \qquad (10)$$

where:

ρ is the resistivity of the ground,

ℓ is the length of the rod,

d is the diameter of the rod,

e_o is the dielectric constant of vacuum, and

e_r is the relative dielectric constant of the soil which varies between 4 and 70 (4 for dry soil, 9 for ordinary moist soil and 70 for distilled water).

2.2 Two or three vertical rods

Another simple grounding system with common use, analysed in this paper, consists of three vertical copper rod as Fig.3 shows.

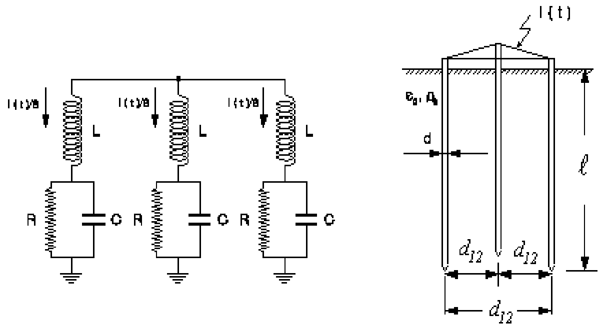

Fig. 3: Equivalent circuit of three vertical rods consisting of compound elements

When the above mentioned system is struck by an impulse current it appears to have a low resistance way to earth with its value decreasing exponentially to that of steady state. The mathematical equation which describes the behaviour of a system consisting of three parallel vertical rods derives from the circuit analysis regarding the elements as compound and in the exact order as shown in Fig.3. The equation is given below:

$$z(t) = \frac{R}{3}\cdot\left[t \cdot \frac{(t-b)\cdot e^{-a\cdot t} - (t-a)\cdot e^{-b\cdot t}}{(t-a)\cdot(t-b)\cdot\left(e^{-a\cdot t} - e^{-b\cdot t}\right)}\right] +$$
$$+ \frac{R}{3}\cdot\left[\frac{(b-a)}{(t-a)\cdot(t-b)}\cdot\frac{e^{-t\cdot t}}{\left(e^{-a\cdot t} - e^{-b\cdot t}\right)}\right] +$$
$$+ \frac{L}{3}\cdot\left[\frac{b\cdot e^{-b\cdot t} - a\cdot e^{-a\cdot t}}{e^{-a\cdot t} - e^{-b\cdot t}}\right] \qquad (11)$$

The stationary resistance of each driven rod is given by the formula [4]:

$$R = \frac{r}{4 \cdot p \cdot \ell}\left(ln\left(\frac{32 \cdot \ell^2}{d \cdot d_{12}}\right) - 2 + \frac{d_{12}}{2 \cdot \ell} - \frac{d_{12}^2}{16 \cdot \ell^2}\right) \qquad (12)$$

The capacitance of each driven rod in an arrangement of three rods at equal distances between each other, is given by [8]:

$$C = \frac{2 \cdot p \cdot e_r \cdot e_o \cdot \ell}{ln\left(\frac{d_{12}}{d}\right)} \qquad (13)$$

The inductance of each driven rod, including the mutual inductance, is given by the following formula [8]:

$$L = 2 \cdot 10^{-7} \cdot p \cdot \ell \cdot \left(ln\left(\frac{1}{r'}\right) + ln\left(d_{12}\right)\right) \qquad (14)$$

where:

r is the radius of the rod,

d_{12} is the distance between each two rods and

$r' = 0{,}7788 \times r$ is the equivalent radius.

In the above formula the equivalent radius *r'* is used in order to take into account the soil ionisation.

3 Results

The coefficients a, b of the double exponential function (Eq.1) have the values of 90909 and 100000 respectively. Table 1 summarises the values of the electrical and physical parameters which were used for the computations.

Rods	ℓ [m]	d [mm]	ρ	e_r	Fig

1	0.5	20	30	4	4
1	1.0	20	30	4	4
1	1.5	20	30	4	4
1	2.0	20	30	4	4
1	1.5	20	10	4	5
1	1.5	20	30	4	5
1	1.5	20	75	4	5
1	1.5	20	100	4	5
1	1.5	5	30	4	6
1	1.5	20	30	4	6
1	1.5	4	30	4	6
1	0-3	20	30	4	7
3	0.5	20	30	10	8
3	1.0	20	30	10	8
3	1.5	20	30	10	8
3	2.0	20	30	10	8
3	1.5	20	10	10	9
3	1.5	20	30	10	9
3	1.5	20	75	10	9
3	1.5	20	100	10	9
3	1.5	5	30	10	10
3	1.5	20	30	10	10
3	1.5	4	30	10	10
3	2	20	0-500	10	11

Table. 1: Electrical and physical parameters of the grounding systems.

The variation of impulse impedance for a vertical rod is presented in Figs.4-7.The variation of impulse impedance for three vertical rods is presented in Figs.8-11. Fig.4, 7 and 8 show that the impulse impedance tends to decrease exponentially with the increase of the electrode length. On the contrary Fig.5, 9 and 11 shows linearity between impulse impedance and soil resistivity. Moreover the variation of the diameter does not affect significantly the impulse impedance.

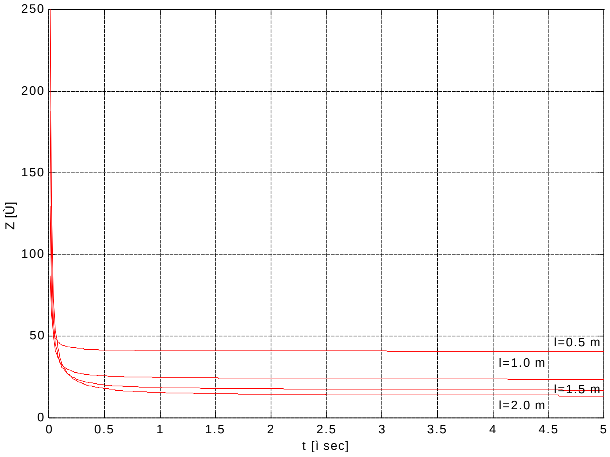

Fig. 4: Impulse impedance vs. time for a driven rod, for various electrode lengths (ρ=30 Ω·m, *d*=20 mm, ℓ=0.5, 1.0, 1.5, 2.0 m)

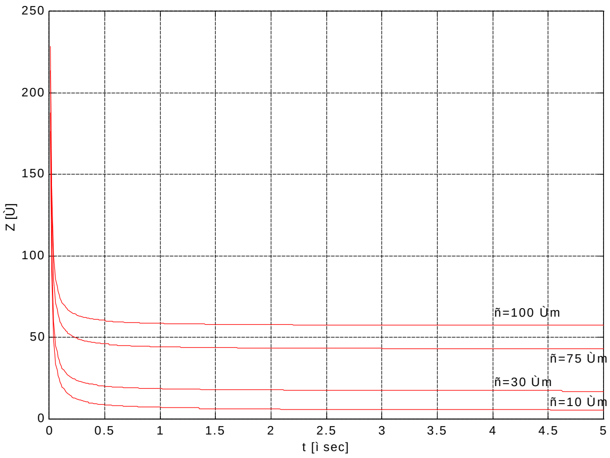

Fig. 5: Impulse impedance vs. time for a driven rod, for various soil resistances (ℓ=1.5 m, *d*=20 mm, ρ=10, 30, 75, 100 Ω·m)

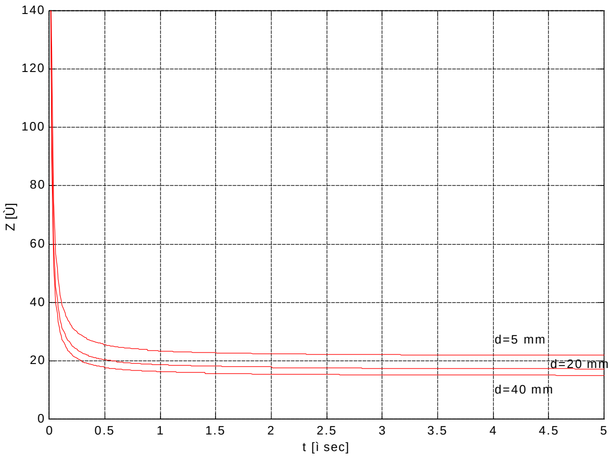

Fig. 6: Impulse impedance vs. time for a driven rod, for various electrode diameters (ℓ=1.5 m, ρ=30 Ω·m, *d*=5, 20, 40 mm)

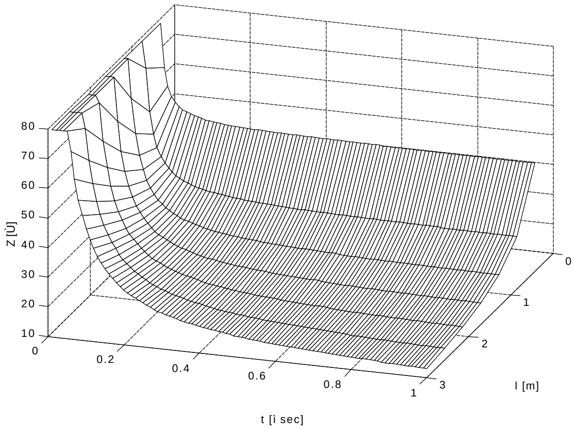

Fig. 7: Impulse impedance vs. time and length for a driven rod, of the rod (d=20 mm, ρ=30 $\Omega\cdot$m)

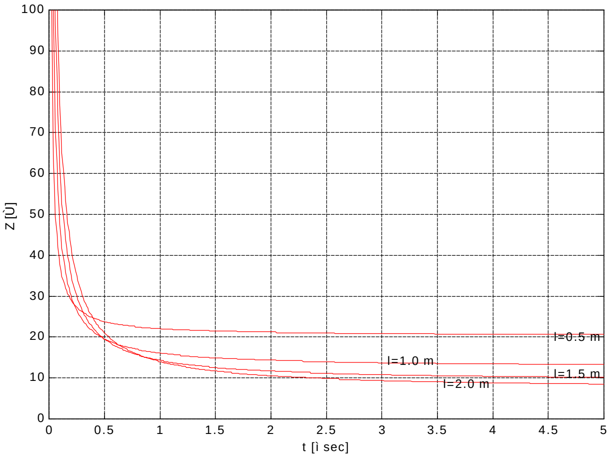

Fig. 8: Impulse impedance vs. time for three vertical rods, for various electrode lengths (ρ=30 $\Omega\cdot$m, d=20 mm, d_{12}=2 m, ℓ=0.5, 1.0, 1.5, 2.0 m)

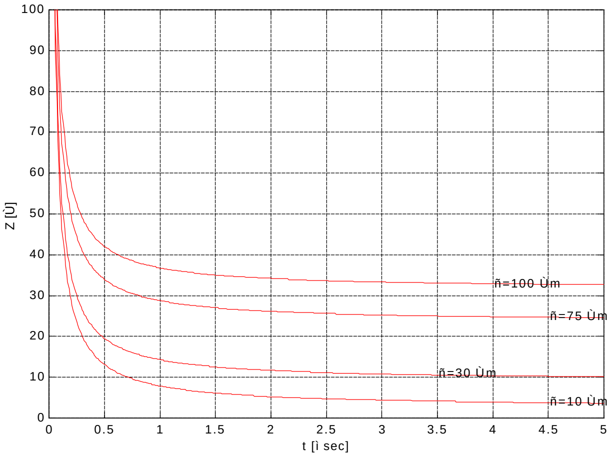

Fig. 9: Impulse impedance vs. time for three vertical rods, for various soil resistances (ℓ=1.5 m, d=20 mm, d_{12}=2 m, ρ=10, 30, 75, 100 $\Omega\cdot$m)

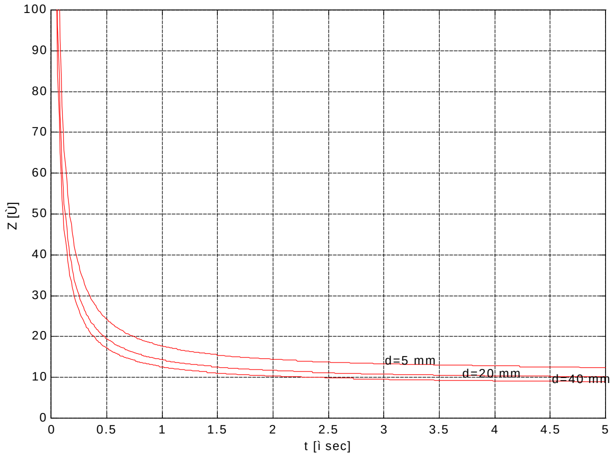

Fig. 10: Impulse impedance vs. time for three vertical rods, for various electrode diameters(ℓ=1.5 m, ρ=30 $\Omega\cdot$m, d_{12}=2 m, d=5, 20, 40 mm)

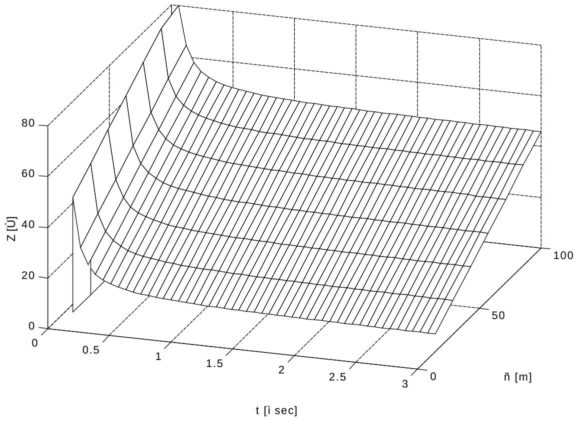

Fig. 11: Impulse impedance vs. time and soil resistance for three vertical rods (d=20 mm, ℓ=2 m, d_{12}=2 m)

4 Conclusions

The optimisation of grounding systems is essential, since their improvement after installation is a difficult task and sometimes not possible. The contribution of this paper is the development of the presented mathematical model which facilitates the calculation of the impulse impedance of ordinary grounding systems from driven rods. The required data for the calculation are only the geometrical and physical characteristics of the rods. The calculated results show that the value of the impulse impedance is quite higher than the one of the steady state (stationary resistance) as it was expected from respective measurements.

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